

# Towards a multi-variable parametric cost model for ground and space telescopes

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## ABSTRACT

Parametric cost models can be used by designers and project managers to perform relative cost comparisons between major architectural cost drivers and allow high-level design trades; enable cost-benefit analysis for technology development investment; and, provide a basis for estimating total project cost between related concepts. This paper hypothesizes a single model, based on published models and engineering intuition, for both ground and space telescopes:

$$\text{OTA Cost} \sim (\mathbf{X}) \mathbf{D}^{(1.75 \pm 0.05)} \lambda^{(-0.5 \pm 0.25)} \mathbf{T}^{-0.25} e^{(-0.04) \mathbf{Y}}$$

Specific findings include: space telescopes cost 50X to 100X more ground telescopes; diameter is the most important CER; cost is reduced by approximately 50% every 20 years (presumably because of technology advance and process improvements); and, for space telescopes, cost associated with wavelength performance is balanced by cost associated with operating temperature. Finally, duplication only reduces cost for the manufacture of identical systems (i.e. multiple aperture sparse arrays or interferometers). And, while duplication does reduce the cost of manufacturing the mirrors of segmented primary mirror, this cost savings does not appear to manifest itself in the final primary mirror assembly (presumably because the structure for a segmented mirror is more complicated than for a monolithic mirror).

**Keywords:** Space Telescope Cost Model, Parametric Cost Model

## 1. INTRODUCTION

Parametric cost models provide several benefits to designers and project managers. They identify major architectural cost drivers and allow high-level design trades; enable cost-benefit analysis for technology development investment; and, they provide a basis for estimating total project cost for budgetary planning and procurement activities. In November 1999, NASA MSFC began an independent cost and schedule analysis of the Optical Telescope Element (OTE) for what would become the Webb Space Telescope. This effort consisted of two thrusts: bottoms-up estimating and top-down parametric modeling. This effort culminated in the November 2000 Re-Scope meeting that changed the JWST mirror collecting area from 50 to 25 square meters, and helped define the procurement “should-cost” estimate.

Subsequently, MSFC continued developing parametric cost models for ground and space telescope assemblies.<sup>[1-12]</sup> Our effort is guided by the principles of being data driven, mathematically rigorous, consistent with engineering practice, mentoring of students and publishing transparency. Our papers fully discuss methodology, including what telescopes are in the data base, what programmatic and engineering data is collected and data completeness; and model creation process, including parameter cross-correlation analysis, sequential variable regression and statistical confidence. This paper hypothesizes a single model for both ground and space telescopes and discusses cost reduction via duplication.

But first, some disclaimers. First, parametric cost models cannot predict the cost of a future system. They are backward looking. They describe how the cost of historical systems varied as a function of cost estimating relations (CERs). The only thing a cost model can do is provide guidance as to how the cost of a potential future system might relate to an existing historical system. Second, a parametric cost model is only as good as its data base. The fundamental challenge of cost modeling is developing a parametric model that includes the most important cost estimating relationships - the most important factors that drive cost. To do this requires a data base with sufficient samples and data diversity to yield statistically meaningful results. And engineering judgment to interpret the results. Given that we are working with a limited sample of telescopes, small additions, corrections and changes to the data base have resulted in statistically significant changes to the models. Third, definitions are important. This model is only for optical telescope assemblies, i.e. the system that collects and concentrates an optical signal, consisting of a primary mirror, secondary and auxiliary optics, and the structure that connects them together. It provides no parametric cost estimating information about science instruments which go behind the telescope nor the spacecraft bus.

## 2. PARAMETRIC OPTICAL TELESCOPE ASSEMBLY COST MODEL

A key objective of our work has been to develop a single model that describes both ground and space telescopes. But, while the ground and space telescope data bases both have good diameter diversity, their diversity does not overlap for other Cost Estimating Relations (CERs). Ground based telescopes have better ‘year of development’ diversity; while space based telescopes have better ‘operating temperature’ diversity. The reason is that over time, the trend on the ground is to make larger aperture telescopes with similar performance specification such as operating temperature and diffraction limit. It is only in space that longer diffraction limited telescopes can be operated at cryogenic temperatures.

Combining the published results for our ground modes (Appendix A) and space models (Appendix B), we hypothesize the following potential multivariable parameter cost model for optical telescope assemblies:

$$\text{OTA Cost} \sim (X) D^{(1.75 \pm 0.05)} \lambda^{(-0.5 \pm 0.25)} T^{-0.25} e^{(-0.04) Y}$$

where:

OTA Cost in Millions of FY2000\$

X = 50 to 100X Space Multiplier

D = Primary Mirror Diameter (meters)

$\lambda$  = Wavelength Diffraction Limited (micrometers)

T = Operating Temperature (Kelvin)

Y = Year of Development – 2000 (years)

Note: this model is not the product of a statistical analysis. It is an engineering intuition extrapolation of our previously published statistical analysis. The justification for this is a belief that, while space telescopes are more expensive than ground telescopes, the basic engineering practices, processes and infrastructure used to make both are nearly identical. And, that space telescopes are more expensive because: (1) mass margin – space telescope have a lower mass margin which requires additional design labor, additional ground support infrastructure, additional testing and more complicated testing, etc.; (2) insight/oversight – space telescope get more managerial and customer attention than ground telescopes.

### 2.1 Year of Development

Year of development (YOD) has been a statistically significant CER for all of our ground models.<sup>[2,10]</sup> We find that the cost of an OTA declines with time (presumably due to technology or process advance) according to  $e^{(-0.04) Y}$ . Interestingly, to date, we have not found YOD to be a significant CER for space telescopes. We have two hypothesis. First, unlike ground telescopes where each new telescope is a variation upon the theme, most space telescopes are unique, one of a kind designs, which require the invention of new technology just to exist. Second, maybe the data for the earliest missions in our data base (OAO-2 and OAO-3) are incomplete and thus masking a YOD trend. A review of the OAO data indicates that the OTA cost could be as much as 50% higher. Also, OAO science instruments were spectroscopic; thus, their optical performance is less demanding (less costly) than for an imaging mission. Analysis of the published record indicates that while the OAO systems were UV instruments, their Point Spread Functions were equivalent to that of an infrared diffraction limited system. We plan to test these hypothesis in future regressions.

We choose to include the  $e^{(-0.04) Y}$  YOD term in our hypothesized general model based on our belief that the same engineering practices apply to both ground and space telescopes. Our belief is supported by both Horak<sup>[13-14]</sup> and Bely<sup>[15]</sup> who published models with a YOD term of  $e^{(-0.033) Y}$ . The practical difference is: an exponent of -0.04 implies that cost reduces by 50% every 17.3 years while an exponent of -0.033 implies a 50% cost reduction every 21 years. To first order, cost probably goes down approximately 50% every 20 years. As an example, the Hubble OTA was initiated in 1977 and, when complete in the mid-1980s, its final cost (in FY11\$) was approximately \$500M.<sup>[12]</sup> By comparison, development of the telescope which would become WFIRST was initiated in the early to mid-1990s (17 to 20 years after Hubble) and when transferred to NASA was assessed to have a cost of approximately \$250M. And, scaling to 2016, the model implies that it should be possible to make a Hubble class OTA for approximately \$125M.

### 2.2 Operating Temperature

Operating temperature is a statistically significant CER for space telescopes but not for ground. The reason is because ground telescopes operate over a narrow ambient temperature range; thus, the ground data has no temperature diversity. But space telescopes operate at temperatures ranging from 300K to 4K. Therefore, we choose to include  $T^{-0.25}$  in the general model. And since cost models are relative, for ground telescopes this term is irrelevant. Additionally, both Horak<sup>[13-14]</sup> and Bely<sup>[15]</sup> models have  $T^{-0.2}$ .

The implication of the temperature term is that a 4K telescope is approximately 2X more expensive than a 77K telescope and approximately 3X more expensive than a 300K telescope. Or, a 50K telescope is about 50% more expensive than a 300K telescope. HOWEVER, the reader must be cautioned. Temperature is coupled with diffraction limited wavelength. Typically, only infrared telescopes require cryogenic temperatures. And, while cryogenic telescopes are more costly, IR telescopes are less costly. These effects tend to cancel. Also, the reader is cautioned, that this CER is only for the OTA. It provides no information about the cost of getting a telescope to or keeping it at a specific operating temperature.

### 2.3 Diffraction Limited Wavelength

Wavelength and aperture diameter are the only two CERs that are statistically significant for both ground and space telescopes. We have published two different ground models with different wavelength dependencies:

**Ground OTA Cost ~ 0.7 SF D<sup>1.8</sup> λ<sup>-0.5</sup> e<sup>-0.04(Y)</sup>** (2005 Stahl and Holmes)

**Ground OTA Cost ~ 0.8 SF D<sup>1.7</sup> λ<sup>-0.7</sup> e<sup>-0.04(Y)</sup>** (2012 Luedtke and Stahl)

The only difference between these two models is the Segmentation Factor (SF). The 2005 model treated each segment prescription as unique, i.e. no learning between prescriptions, while the 2012 model assumes learning. The most significant difference between the two is the wavelength. But, given that the ground data base for both models were essentially identical and that there is limited wavelength diversity in the data base (i.e. all ground telescopes in the data base are in the 0.5 to 10 micrometer range), the -0.7 exponent may or may not be more correct than the -0.5 exponent.

Adding to the uncertainty, our published multivariable space model has a different wavelength dependence:

**Space OTA Cost ~ \$144M D<sup>1.7</sup> λ<sup>-0.3</sup> T<sup>-0.25</sup>** (2012 Stahl and Henrichs)

Additionally, the Horak<sup>[13-14]</sup> and Bely<sup>[15]</sup> models have λ<sup>(-0.18)</sup>; and, Kahan<sup>[16]</sup> asserted that cost scales with λ<sup>(-0.5)</sup>.

The only way to definitively resolve this problem is to get more wavelength diverse data. One source of semi-quantitate data is Meinel's papers. A comparison of optical and radio wavelength telescopes indicates approximately a 100X to 1000X difference in cost.<sup>[17,18]</sup> Assuming that an optical telescope operates at a wavelength of 1 micrometer and that radio telescopes operate from 10 mm to 10 meters, an exponent of -0.5 implies that a radio telescope will cost 100X to 3000X less while an exponent of -0.7 implies a 600 to 80,000X cost reduction and an exponent of -0.3 implies a 15 to 125X reduction. Similarly, a comparison of optical and submillimeter telescopes indicates a 10X cost difference.<sup>[19]</sup> Assuming that the submillimeter wavelength is 100 micrometers to 1 mm, then a -0.5 exponent indicates a 10X to 30X cost reduction while a -0.7 exponent indicates a 25 to 125X reduction and -0.3 implies 4 to 8X reduction. Thus, for right now, we are selecting λ<sup>(-0.5 ± 0.25)</sup>. And, we are seeking radio and microwave telescopes for the database.

Some examples assuming that the range for this exponent is -0.25 to -0.5. The Webb Telescope OTA is diffraction limited at 2 micrometers. It will likely cost 1.4 to 2X more to make a JWST OTA diffraction limited at 500 nm. But, as discussed in Section 2.2, this cost increase would be offset by a 1.8X cost decrease in changing the operating temperature from 30K to 300K. Alternatively, it will likely cost 2.2 to 5X less to make a JWST OTA diffraction limited at 50 micrometers, but this savings would be offset by 1.3X increase in going from 30K to 10K. Again, the reader is cautioned, these CERs only related to the telescope. They do not provide information about the cost of getting a telescope to or keeping it at a specific operating temperature.

Finally, engineering intuition implies that there may really is a wavelength CER difference between ground and space telescopes because of gravity and wind. Ground telescopes must be designed to provide a stable wavefront in a gravity field and under exposure to wind. Space telescopes have neither. Thus, maybe the cost of ground telescopes really do have a greater dependence on wavelength performance than space telescopes.

### 2.4 Aperture Diameter

Diameter CER is remarkably consistent for all of our multivariable Ground and Space models (varying between 1.7 and 1.8). Therefore, we are selecting D<sup>(1.75 ± 0.05)</sup>. Additionally, it is similar to Bely's model. But, it is different from Horak's diameter exponent of 0.7 – whose data base consisted mostly of strategic and experimental IR sensors.<sup>[4]</sup>

<b>Ground OTA Cost</b>	<b>~ \$0.7M SF D<sup>1.8</sup> λ<sup>-0.5</sup> e<sup>-0.04(Y)</sup></b>	<b>(2005 Stahl and Holmes)</b>
<b>Ground OTA Cost</b>	<b>~ \$0.8M SF D<sup>1.7</sup> λ<sup>-0.7</sup> e<sup>-0.04(Y)</sup></b>	<b>(2012 Luedtke and Stahl)</b>
<b>Space OTA Cost</b>	<b>~ \$144M D<sup>1.7</sup> λ<sup>-0.3</sup> T<sup>-0.25</sup></b>	<b>(2012 Stahl, Henrichs, Luedtke and West)</b>
<b>OTA Cost</b>	<b>~ D<sup>1.6</sup> λ<sup>-0.18</sup> T<sup>-0.2</sup> e<sup>-0.033(Y)</sup></b>	<b>(2003 Bely)</b>

Some examples assuming an exponent of 1.75. As discussed in Section 2.1, the current estimated cost of a 2.4-meter HST class OTA is approximately \$125M. If this telescope were scaled to 4-meters, its likely cost would be approximately \$300M. Or, the cost of the 6.5-meter Webb OTA is \$1.2B. If this telescope were scaled to 13-meters, its likely cost would be close to \$4B. Of course, as discussed in Section 2.1, these costs will tend to reduce with time as a consequence of technology development and process improvements.

2.5 Ground to Space Multiplier

Because we intend our cost models to be used to estimate relative cost and not to predict actual cost, we have not previously published the leading coefficients for our models. Also, these leading coefficients can vary significantly from model to model. But, if the goal is to develop a generic model, then a space cost multiplier is needed.

To develop a space multiplier, it is necessary to compare (as best as possible) apples to apples regressions. While diameter only models do not adequately explain the variation of cost in our data bases, they are easy to generate and we have published several diameter models for both ground and space telescopes:

Ground	\$ 3M D <sup>1.4</sup>	(2013Stahl and Henrichs)
Ground	\$ 2M D <sup>1.4</sup>	(2012 Luedtke and Stahl)
Ground	\$ 2M D <sup>1.4</sup>	(2012 Stahl, Henrichs, Luedtke and West)
Space	\$100M D <sup>1.4</sup> (N=15)	(2012 Stahl, Henrichs, Luedtke and West)
Space	\$30M D <sup>1.4</sup> (N=15)	(2013 Stahl and Henrichs)
Space	\$38M D <sup>1.6</sup> (N=13)	(2013 Stahl and Henrichs)
Space	\$45M D <sup>2.0</sup> (N=12)	(2013 Stahl and Henrichs)

These models imply that space is 15X to 50X more costly than ground.

Now consider our multivariable models – again, we never previously published the leading coefficients.

Ground OTA Cost	~ \$0.7M SF D <sup>1.8</sup> λ <sup>-0.5</sup> e <sup>-0.04(Y)</sup>	(2005 Stahl and Holmes)
Ground OTA Cost	~ \$0.8M SF D <sup>1.7</sup> λ <sup>-0.7</sup> e <sup>-0.04(Y)</sup>	(2012 Luedtke and Stahl)
Space OTA Cost	~ \$144M D <sup>1.7</sup> λ <sup>-0.3</sup> T <sup>-0.25</sup>	(2012 Stahl, Henrichs, Luedtke and West)
Space OTA Cost	~ \$42M D <sup>1.5</sup> λ <sup>-0.24</sup> e <sup>-0.01(Y)</sup>	(previously unpublished from 2012)

These models imply that space is 50X to 200X more costly than ground.

Finally, the following analysis implies that space telescopes are 300X more expensive than ground. One explanation for the cost multiplier is mass margin. Mass margin drives design which drives cost (Stahl, et. al., 2013<sup>[12]</sup>). Space telescopes typically have a mass margin on the order of 1.4X and primary mirror areal densities range from 70 to 300 kg/m<sup>2</sup>. Ground telescopes typically have a mass margin on the order of 2 to 3X and their primary mirror areal densities range from 200 to 1000 kg/m<sup>2</sup>. At just the primary mirror level, ground based telescopes may be 3X more massive than space telescopes. Figure 1 plots cost density as a function of diameter for ground, space and attached telescopes. All ground telescopes have approximately the same cost per kg. And, all space telescopes also have approximately the same cost per kg. But these costs per kg differ by 1000X – implying that maybe space telescopes are 300X more expensive than ground.

Defining a space cost multiplier is difficult. But, given the range of our results, we choose to say that space telescopes are 50X to 100X more costly than ground telescopes.

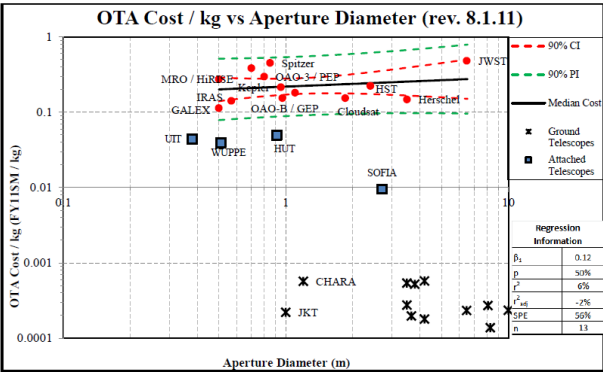


Figure 1: Cost per kg of Space Free-Flying, Space Attached, Airborne and Ground indicates that different design rules apply for each class of telescope.

### 3. SEGMENTATION OR DUPLICATION FACTOR

While a statistically significant CER, the segmentation or duplication factor is probably the most misunderstood and misapplied aspect of our modeling. Its original motivation was to justify (on behalf of the JWST program) an assertion that a segmented aperture telescope will have a lower cost than a monolithic aperture telescope. The goal of the early JWST development program was to ‘break the cost curve’. This assertion was based on published data from the manufacture of the VLT and Gemini mirrors<sup>[2]</sup> (Figure 2) and unpublished results from Chandra/AXAF mirror polishing. Subsequently, results from polishing the JWST primary mirror segment assemblies supported this assertion.<sup>[10]</sup>

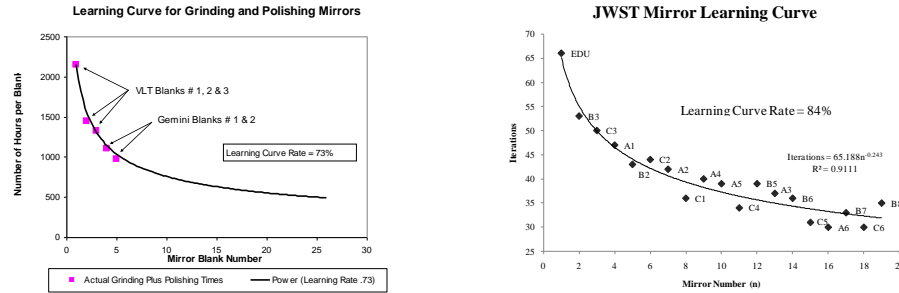


Figure 2: Learning Curve for fabricating 5 8-meter primary mirrors for VLT & Gemini is 73%; Learning curve for fabricating 19 1.5-meter segments for JWST is 84% (private communication, Ben Gallagher).

The challenge in developing a segmentation factor CER was that there had been no segmented aperture space telescopes. And, the number of segmented aperture ground telescopes was limited, so we included multi-aperture systems. Consequently, our results depend on five telescopes: CHARA, LBT, HET, Keck 1 and Keck 2.

In 2005, Stahl et. al. published a multivariable ground model that included a primary mirror segmentation factor.<sup>[2]</sup>

$SF = P_n R_n^{0.7} (Ds/D)^{1.8}$  if the primary mirror is segmented, and

$SF = 1$  if the primary mirror is monolithic

where:  $D_s$  = Diameter of the Repeated Segments (meters)

$D$  = Diameter of the Primary Mirror (meters)

$R_n$  = Number of Repeated Segments

$P_n$  = Number of Unique Prescriptions

The purpose of the Segmentation Factor was to quantify cost reduction from learning while making duplicate primary mirror segments (such as for Keck or HET) or whole primary mirrors for arrays of telescopes (such as for VLT or CARA). The 0.7 exponent is an 80% learning curve (where each successive unit is 80% the cost of the previous unit) and was consistent with the published REOSC and unpublished CHANDRA/AXAF results. In this regression, we assumed that the different segments or complete OTAs were identical and that learning was confined to a given prescription or implementation.

Luedtke and Stahl 2012<sup>[10]</sup>, published a simplified (but functionally identical) segmentation factor (based on the same 5 segmented or sparse aperture telescopes) which assumed that learning transfers across prescriptions:

$$SF = [S^{0.7} (Ds/D)^{1.7}]$$

Where  $S$  = total number of segments.

The problem is that these factors ONLY describe the cost reduction associated with making exact duplicates – and henceforth shall be called Duplication Factors (DF). The DF actually describes the cost reduction associated with making two or more of the exact same telescope for a sparse aperture array or interferometer. What the DF captures is the cost reduction of making a second Keck or the cost reduction of making a CARA array or an LBT. And, while the DF is consistent with the cost reduction shown in making the JWST segments, this cost reduction does not appear to extend to the telescope itself. Segmenting the JWST primary mirror did not ‘break the cost curve’. The cost of JWST’s primary mirror assembly is ‘in family’ for a 6.5-m diameter aperture telescope. Yes, JWST demonstrated cost reduction in the manufacture of 18 1.5-m class segments (estimated at 63% of cost of a 6.3-m monolith). But, these segments need to be mounted to a structure. And, the complexity (and cost) of a structure needed to support a segmented primary mirror is greater than for a monolithic mirror.

## 4. CONCLUSIONS

Parametric cost models provide several benefits to designers and project managers. They identify major architectural cost drivers and allow high-level design trades; enable cost-benefit analysis for technology development investment; and, they provide a basis for estimating total project cost for budgetary planning and procurement activities.

But, their use requires caution. First, parametric cost models cannot predict the cost of a future system. They are backward looking. They describe how the cost of historical systems varied as a function of cost estimating relations (CERs). The only thing a cost model can do is provide guidance as to how the cost of a potential future system might relate to an existing historical system. Second, a parametric cost model is only as good as its data base. The fundamental challenge of cost modeling is developing a parametric model that includes the most important cost estimating relationships - the most important factors that drive cost. To do this requires a data base with sufficient samples and data diversity to yield statistically meaningful results. And engineering judgment to interpret the results. Given that we are working with a limited sample of telescopes, small additions, corrections and changes to the data base have resulted in statistically significant changes to the models. Third, definitions are important. This model is only for optical telescope assemblies, i.e. the system that collects and concentrates an optical signal, consisting of a primary mirror, secondary and auxiliary optics, and the structure that connects them together. It provides no parametric cost estimating information about science instruments which go behind the telescope nor the spacecraft bus.

This paper hypothesizes a single model that describes both ground and space telescopes:

$$\text{OTA Cost} \sim (X) D^{(1.75 \pm 0.05)} \lambda^{(-0.5 \pm 0.25)} T^{-0.25} e^{(-0.04) Y}$$

where:

OTA Cost in Millions of FY2000\$

X = 50 to 100X Space Multiplier

D = Primary Mirror Diameter (meters)

$\lambda$  = Wavelength Diffraction Limited (micrometers)

T = Operating Temperature (Kelvin)

Y = Year of Development – 2000 (years)

This model is not the product of a statistical analysis. It is engineering intuition extrapolation of our previously published statistical analysis. The justification for this composite model is a belief that, while space telescopes are more expensive than ground telescopes, the basic engineering practices, processes and infrastructure used to make both are nearly identical. And, that space telescopes are more expensive because: (1) mass margin – space telescope have a lower mass margin which requires additional design labor, ground support infrastructure, and testing; (2) insight/oversight – space telescope get more managerial and customer attention than ground telescopes.

Specific implications of the cost model include: space telescopes are 50X to 100X more costly than ground telescopes; diameter is the most important CER; cost is reduced by approximately 50% every 20 years (presumably because of technology advance and process improvements); and, for space telescopes, cost associated with diffraction limited wavelength is balanced by cost associated with operating temperature. For example, based on year of development, scaling HST's \$500M cost (FY11) from its development in the late 1970s to the mid 1990's (when the WFIRST/AFTA telescope was developed) predicts that a HST class telescope should cost \$250M (the cost for which it was transferred to NASA). Scaling to current time predicts that a HST class telescope might cost as little as \$125M. Now, scaling that telescope to 4-meter predicts a potential cost of about \$300M. Another example, ignoring wavelength and operating temperature (which tend to cancel), scaling HST approximately 25 years from late 1970s to early 2000s implies a base cost for a 2.4-m telescope of close to \$200M. Scaling this from 2.4-m to 6.5-m implies a cost of \$1B to \$1.1B (FY11) which with inflation is close to the JWST actual OTA cost of \$1.2B. And a third example, scaling JWST's OTA to 13-meters implies a potential cost of \$4B. Also, all future costs are likely to be lower due to year of development.

Finally, Duplication Factor only reduces cost when identical systems are being manufacture, i.e. for multiple aperture sparse arrays or interferometers. While Duplication Factor does reduce the cost of a manufacturing the mirrors of segmented primary mirror over that of a monolithic mirror, this cost savings does not appear to manifest itself in the final primary mirror assembly. We speculate that this is because the structure for a segmented mirror is more complicated than for a monolithic mirror.

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## APPENDIX A: GROUND OPTICAL TELESCOPE ASSEMBLY COST MODEL

### A.1 Single Variable Ground Optical Telescope Assembly Cost Models

In 2005, Stahl et al. published a single variable model which estimated cost as a function of primary mirror diameter.<sup>[2]</sup>

$$\text{Ground OTA Cost} \propto \text{Diameter}^{1.8}$$

As discussed in Luedtke and Stahl, 2012, this model was wrong because it excluded the leading multiplier from the regression. Doing this forced the cost estimate of small aperture telescopes towards 1; which resulted in a ‘steeper’ cost curve with diameter. Also, by omitting the leading multiplier, the regression was not independent to units, i.e. a different exponent would have been returned if the regression had been done in centimeters instead of meters. The corrected model was published in 2012:

$$\text{Ground OTA Cost} \sim \$2\text{M} \times \text{Diameter}^{1.4}$$

By including the leading multiplier, a ‘flatter’ cost curve with diameter that is independent of units was obtained.

One reason that the 2005 paper published the single variable model without the multiplier is because the coefficient was similar with that of the multi-variable model, but in fact it should not have matched. The reason why the single variable diameter exponent should be lower than the multivariable diameter exponent is because of the effect of year of development. Year of development and aperture diameter are positively correlated ( $r=0.55$ ,  $p=0.03$ ), a fact which makes sense because one tends to build the largest telescope that can be afforded at any given time. Technology advancement tends to reduce the cost to fabricate a telescope of a given aperture diameter, when year of development is implicitly included in the single variable model, it results in a smaller diameter exponent. But, when year of telescope development is made explicit as a cost variable independent of aperture diameter, it results in a larger diameter exponent.

### A.2 Multivariable Ground Optical Telescope Assembly Cost Models

Also in the 2005 paper, Stahl et. al. published a multivariable model which estimated cost as a function of primary mirror diameter, diffraction-limited wavelength, and year of development.<sup>[2]</sup> The multi-variable model also included a factor for primary mirror segmentation and/or duplication:

$$\text{Ground OTA Cost} \sim 0.7 \text{ SF } D^{1.8} \lambda^{-0.5} e^{-0.04(Y)}$$

$$(R^2=95\%, \text{ adjusted } R^2=94\%, \text{ SPE} = 35\%)$$

Where:

OTA Cost in Millions of FY2000\$

D = Primary Mirror Diameter (meters)

$\lambda$  = Wavelength Diffraction Limited (microns)

Y = Year of Development - 2000

SF = Segmentation Factor

SF =  $P_n R_n^{0.7} (D_s/D)^{1.8}$  if the primary mirror is segmented, and  
SF = 1 if the primary mirror is monolithic

Where

$D_s$  = Diameter of the Repeated Segments (meters)

D = Diameter of the Primary Mirror (meters)

$R_n$  = Number of Repeated Segments

$P_n$  = Number of Unique Prescriptions

The purpose of the Segmentation Factor was to quantify cost reduction from learning while making duplicate primary mirror segments (such as for Keck or HET) or whole primary mirrors for arrays of telescopes (such as for VLT or CARA). The data base used to derive the above result contained only 5 segmented monolithic or sparse aperture telescopes. The 0.7 exponent is an 80% learning curve (where each successive unit is 80% the cost of the previous unit). And, in all cases the duplicated segments or complete OTAs were assumed to be identical. Therefore, an assumption was made that learning was confined to a given prescription or implementation. However, results from the JWST primary mirror assembly segments (PMSAs) indicate that learning transfers across prescriptions (Figure A-1).



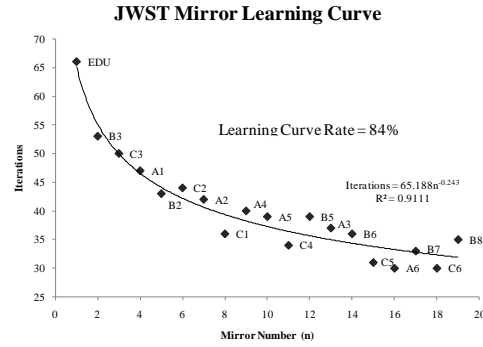
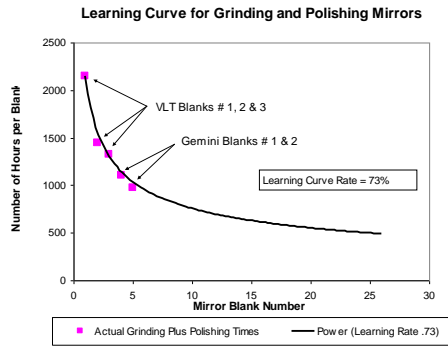


Figure A-1: Learning Curve for fabricating 5 8-meter primary mirrors for VLT & Gemini is 73%; Learning curve for fabricating 19 1.5-meter segments for JWST is 84% (private communication, Ben Gallagher).

Given the JWST results, Luedtke and Stahl 2012, published a new multivariable parametric ground telescope cost model. The model is derived from a data set with the same 5 segmented or sparse aperture telescopes and assumes that learning transfers across prescriptions:

$$\text{Ground OTA Cost} \sim 0.8 \text{ SF } D^{1.7} \lambda^{-0.7} e^{-0.04(Y)}$$

$$(R^2=91\%, \text{ adjusted } R^2=88\%, \text{ SPE} = 37\%)$$

where  $\text{SF} = [S^{0.7} (D_s/D)^{1.7}]$  and  $S$  represents the total number of segments. While the duplication and diameter exponents are nearly identical, the wavelength exponent is significantly different. However, given the size and similarity of the current data base, the new -0.7 exponent may or may not be more correct than the old -0.5 exponent. The problem is one of insufficient wavelength diversity in the data base. While multiple authors have given rules of thumb for how cost varies with wavelength,<sup>[4,16]</sup> only Meinel's papers contain quantitative data.<sup>[4]</sup> A comparison of optical and radio wavelength telescopes indicates approximately a 100X to 1000X difference in cost.<sup>[17,18]</sup> Assuming that an optical telescope operates at a wavelength of 1 micrometer and that radio telescopes operate from 10 mm to 10 meters, an exponent of -0.5 implies that a radio telescope will cost 100X to 3000X less while an exponent of -0.7 implies a 600 to 80,000X cost reduction. Similarly, a comparison of optical and submillimeter telescopes indicates a 10X cost difference.<sup>[19]</sup> Assuming that the submillimeter wavelength is 100 micrometers to 1 mm, then a -0.5 exponent indicates a 10X to 30X cost reduction while a -0.7 exponent indicates a 25 to 125X reduction.

## APPENDIX B: SPACE OPTICAL TELESCOPE ASSEMBLY COST MODEL

Developing a parametric model for space telescopes has been an iterative process driven by evolving database knowledge. From 2009 to 2010, our group published a series of papers.<sup>[3,5-8]</sup> In September 2010, our database underwent an independent review by the NRO Cost Model Office and, while they did not provide us with access to their database or give us any specific data, they did identify specific missions of concern. In response, we systematically reviewed all missions in our database. As a result, some were removed and others were revised. These revisions ranged from slight to dramatic. Additionally, we added a few new mission. Subsequently, we published three papers with corrected findings.<sup>[9, 11-12]</sup> While some of the space telescope database has changed slightly since 2011, this changes have not significantly changed the models. However, we continue to seek new data and refine our interpretation of it.

### B.1 Single Variable Space Optical Telescope Assembly Cost Models

Stahl et al. [2013] published a single variable model that estimates cost as a function of primary mirror diameter: <sup>[12]</sup>

$$\text{Space OTA Cost} \sim \$30M D^{1.4} (N = 15; r^2 = 81\%; \text{SPE} = 122)$$

This model is based on 15 free-flying space telescopes, accounts for 81% of the cost variation in the real data, but is noisy - thus indicating the need for a multi-variable model. As will be discussed below, adding more variables increases fit confidence and reduces noise.

To illustrate the sensitivity of cost modeling to which missions are included or excluded. Excluding CloudSat, Planck and Herschel from the regression yields:

$$\text{Space OTA Cost} \sim \$45\text{M D}^{2.0} \quad (N = 12; r^2 = 94\%; SPE = 62)$$

And, adding Herschel back yields:

$$\text{Space OTA Cost} \sim \$38\text{M D}^{1.6} \quad (N = 13; r^2 = 83\%; SPE = 89)$$

## B.2 Multivariable Space Optical Telescope Assembly Cost Models

Given that the single variable aperture model accounts for only 82% of the actual OTA cost variation and is noisy, a multivariable model is required. Stahl et. al. [2012] published two potential multivariable models. The first estimates cost as a function of primary mirror diameter and diffraction limited wavelength:<sup>[11]</sup>

$$\text{OTA Cost} \sim \text{Dia}^{1.6} \lambda^{-0.2} \quad (N = 12, r^2 = 98\%; SPE = 60\%)$$

The second adds operating temperature:<sup>[11]</sup>

$$\text{OTA Cost} \sim \$144\text{M D}^{1.7} \lambda^{-0.3} T^{-0.25} \quad (N = 11, r^2 = 96\%; SPE = 54\%)$$

This model estimates 96% of the cost variation and is less noisy.

Two comments about wavelength and temperature. First, they are negatively correlated for space telescopes. The longer the operating wavelength, the lower the operating temperature. Second, the sign and magnitudes of the wavelength and temperature exponents tend to balance each other.

Interestingly, Year of Development (YOD) does not seem to be a CER for space telescopes. This is curious given that YOD is important for ground telescopes. So, why does technology advance tend to reduce cost as a function of time for ground telescopes but not for space telescopes? One explanation is that unlike ground telescopes, where each new telescope is a variation upon the theme, most space telescopes are not. Most space telescopes are unique, one of a kind designs which require the invention of new technology just to exist. Another explanation that maybe the data for the earliest missions in our data base (OAO-2 and OAO-3) are incomplete. We are reviewing the data.

## B.3 Optical Telescope Assembly Cost as a percentage of Total Mission Cost

Based on an analysis of detailed work breakdown structures for 7 of the 14 free flying UVOIR space telescope missions in the data base for which we have both OTA and Total Mission cost, the average OTA cost is only 12% of total mission cost (Figure B-1). For these missions, the spacecraft and science instruments account for half of the total mission cost, and the OTA cost is comparable with Program Management and Systems Engineering cost. It must be stated that the 12% OTA cost is an average. The actual costs of OTA as a function of Total, for all 14 missions for which we have data, varies from 2 to 20% (Figure B-2).<sup>[11-12]</sup>

Additionally, Observatory level I&T cost is only 10% of total mission cost. However, in this accounting, mission I&T cost excludes sub-system level I&T cost. Given that sub-system I&T costs might be 10 to 25% of their individual costs, it is plausible that total I&T costs might be as large as 25% of total mission cost.

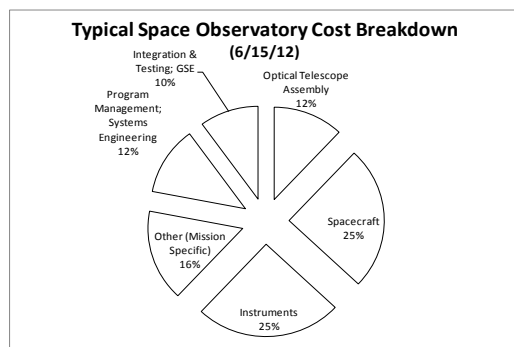


Figure B-1: Average Percentage of Total Mission Cost for Major Cost Elements and/or Sub-Systems.

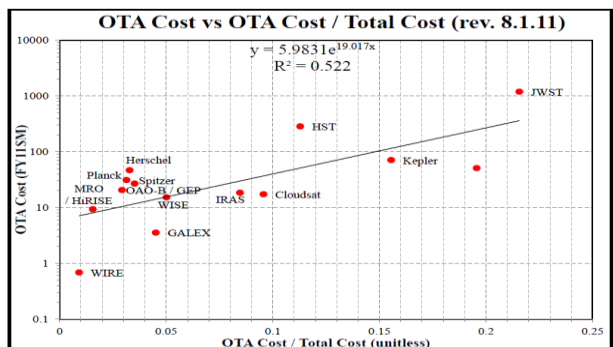


Figure B-2: OTA Cost as a function of Total Mission Cost